ADVANCED MATH / 01 UNDERSTAND

Vertex form

An illustrated lesson ▪ Theory, reasoning and worked examples

The vertex is a turning point

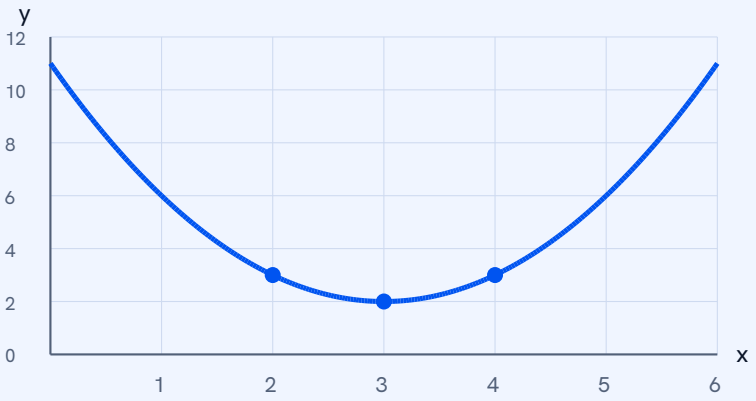
In $y = a(x - h)^2 + k$, the vertex is (h, k). If $a > 0$, the graph opens upward and its vertex is a minimum. If $a < 0$, it opens downward and its vertex is a maximum. The vertical line $x = h$ is the axis of symmetry.

Why the form works

A square is never negative for a real input, and $(x - h)^2$ equals zero exactly when $x = h$. For positive a, every other input adds a positive amount to k. For negative a, every other input subtracts from k. The extremum follows from this property of squares, not from a memorized sign trick.

Read shifts carefully

In $(x - 3)^2 + 2$, the horizontal shift is 3 to the right and the vertical shift is 2 upward. In $(x + 3)^2 + 2$, rewrite $x + 3$ as $x - (-3)$: the vertex moves left to $x = -3$. The input that makes the square zero is the reliable way to read h.



$y = (x - 3)^2 + 2$

Vertex: (3, 2)

Minimum output: 2

Equal distance from 3 gives equal output.

Illustrative teaching diagram. Values and examples are explained in the lesson.

THE ESSENTIAL IDEA

In $y = a(x - h)^2 + k$, the vertex is (h, k).

ADVANCED MATH / 02 BUILD THE METHOD

From idea to method

Convert from standard form

For $ax^2 + bx + c$, the vertex input is $h = -b/(2a)$. Substitute h into the function to obtain k. Completing the square gives the same result while explicitly rewriting the whole function in vertex form. Do not confuse the input h with the minimum or maximum output k.

Understand the scale factor

A larger absolute value of a makes the graph narrower relative to $y = x^2$ on the same axes. Moving one unit from h changes y by a; moving two units changes it by 4a. This squared growth explains the shape and provides quick points for a sketch.

Respect the domain of a model

A parabola used for height over time may only apply between launch and landing. A quadratic cost model may only permit nonnegative quantities. If the vertex lies outside a restricted interval, the greatest or least permitted value may occur at an endpoint instead.

WORKED EXAMPLE / FIND A MAXIMUM

A model is $H(t) = -2(t - 3)^2 + 20$. What is its maximum value and when does it occur?

01 Set up the reasoning

The coefficient -2 is negative, so the parabola opens downward.

02 Apply the relationship

The squared term is zero at $t = 3$. Then $H(3) = 20$.

03 Interpret and check

At $t = 2$ and $t = 4$, $H = 18$. Equal distances from 3 give equal outputs, confirming symmetry.

Keep these distinctions handy

Vertex form	$a(x - h)^2 + k$ exposes the vertex (h, k).
Standard form	$ax^2 + bx + c$ gives $h = -b/(2a)$; find k by substitution.
Opening direction	$a > 0$: minimum. $a < 0$: maximum.

ADVANCED MATH / 03 CONNECT & REMEMBER

See the deeper pattern

Recover vertex form by completing the square

For $x^2 - 6x + 11$, group $x^2 - 6x$ and add and subtract 9: $(x^2 - 6x + 9) + 2 = (x - 3)^2 + 2$. The added 9 completes a square; subtracting the same amount preserves the expression. The vertex is (3, 2), and the function has no real x-intercepts because its minimum is positive.

ONE MORE WORKED APPLICATION

The function $f(x) = 8(x - 7)^2 + 13$ has a minimum at $x = h$. What is h?

1. A square is nonnegative and 8 is positive.

2. The square is zero when $x = 7$, giving the minimum $f(x) = 13$.

Answer: 7

Mistakes to recognize

- The minimum value is k, not h.
- A negative a changes a minimum into a maximum.
- An x-intercept is generally not the vertex.

A quick understanding check

Where is the vertex of $2(x + 4)^2 - 1$?

Explanation: At (-4, -1), because $x = -4$ makes the square zero.

MORE PRACTICE IN YOUR DASHBOARD

Open prepvora.com/app/ and use the SAT question bank. Work on Advanced Math, then look for vertex form questions. Solve independently before reviewing the explanation. Use this guide for the theory; use the dashboard for a longer practice session.

Open your dashboard → | Explore the resource library →

Original Prepvora explanations and examples. Scope: this topic, not the entire SAT syllabus. 2026-10-01 ▪ theory-v2