ADVANCED MATH / 01 UNDERSTAND

Quadratic roots

An illustrated lesson ▪ Theory, reasoning and worked examples

Roots are inputs that produce zero

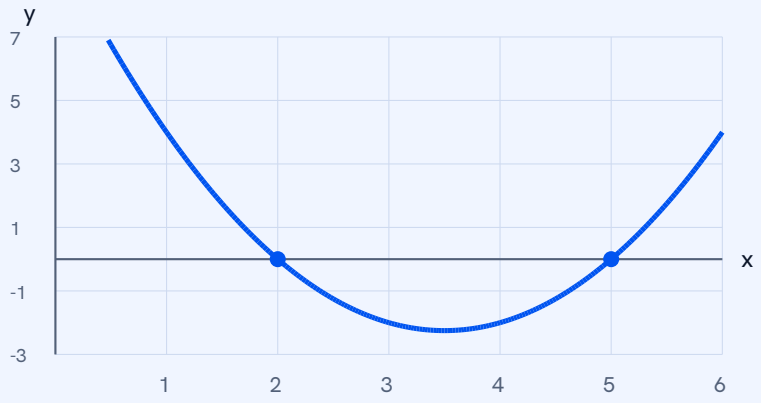
For a quadratic $f(x) = ax^2 + bx + c$ with a nonzero, roots are values of x for which $f(x) = 0$. On a graph, real roots are x-intercepts. The root is an input value, while the corresponding point is (x, 0). A quadratic can have two, one repeated, or no real roots.

The zero-product principle

A product is zero only if at least one factor is zero. Therefore $(x - 2)(x - 5) = 0$ gives $x = 2$ or $x = 5$. This reasoning works because the entire product equals zero. It does not justify setting each factor to zero when the product equals another number.

Different forms expose different facts

Factored form shows roots; vertex form shows the turning point; standard form shows coefficients and the y-intercept. Rewriting does not change the function. Choose a form that makes the requested feature visible instead of expanding or factoring automatically in every problem.



$y = (x - 2)(x - 5)$

Roots: 2 and 5

x-intercepts:
(2, 0) and (5, 0)

Symmetry: x = 3.5

Illustrative teaching diagram. Values and examples are explained in the lesson.

THE ESSENTIAL IDEA

Factoring rewrites a quadratic as a product whose zeroes are easy to read.

ADVANCED MATH / 02 BUILD THE METHOD

From idea to method

Factor with a product and a sum

For $x^2 - 7x + 10$, seek two numbers whose product is 10 and whose sum is -7. They are -2 and -5, producing $(x - 2)(x - 5)$. Expand your factors to verify the middle term as well as the constant; matching only the constant is insufficient.

Use the discriminant when roots are hidden

For $ax^2 + bx + c = 0$, $D = b^2 - 4ac$ determines the number of real roots: positive gives two, zero gives one repeated root, and negative gives none. The quadratic formula $x = (-b \pm \sqrt{(D)})/(2a)$ gives the values when factoring is inconvenient.

Connect symmetry with roots

When two real roots are r and s, the axis of symmetry is $x = (r + s)/2$. The turning point lies halfway between the intercepts horizontally. Its y-coordinate requires substitution; it is not the average of the roots. The coefficient a determines whether the graph opens upward or downward.

WORKED EXAMPLE / A REPEATED ROOT

How many real solutions does $x^2 - 6x + 9 = 0$ have?

01 Set up the reasoning

Factor: $x^2 - 6x + 9 = (x - 3)^2$.

02 Apply the relationship

The only input making the square zero is $x = 3$. It is a repeated root, not two different solutions.

03 Interpret and check

The discriminant is $(-6)^2 - 4(1)(9) = 0$, confirming the graph touches the x-axis once.

Keep these distinctions handy

$D > 0$	Two distinct real roots; the graph crosses the x-axis twice.
$D = 0$	One repeated real root; the graph touches the x-axis.
$D < 0$	No real roots; the graph does not meet the x-axis.

ADVANCED MATH / 03 CONNECT & REMEMBER

See the deeper pattern

Root information without solving

The sum of the roots is -b/a and their product is c/a when roots are counted with multiplicity. For $x^2 - 7x + 10$, the sum is 7 and the product is 10. These relationships can answer a question directly, but the discriminant is still needed if you must establish whether the roots are real.

ONE MORE WORKED APPLICATION

What is the larger solution of $(x - 6)(x - 15) = 0$?

1. The factors give $x = 6$ or $x = 15$.

2. The larger solution is 15.

Answer: 15

Mistakes to recognize

- A factor $x + 4$ gives a root of -4.
- Set the full equation equal to zero before using the zero-product principle.
- A squared factor gives one distinct root with multiplicity two.

A quick understanding check

Does $(x - 2)(x - 5) = 6$ have roots 2 and 5?

Explanation: No. Those values make the product zero, not 6. Rearrange the equation first.

MORE PRACTICE IN YOUR DASHBOARD

Open prepvora.com/app/ and use the SAT question bank. Work on Advanced Math, then look for quadratic roots questions. Solve independently before reviewing the explanation. Use this guide for the theory; use the dashboard for a longer practice session.

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