ALGEBRA / 01 UNDERSTAND

Linear functions

An illustrated lesson ▪ Theory, reasoning and worked examples

Constant change defines a line

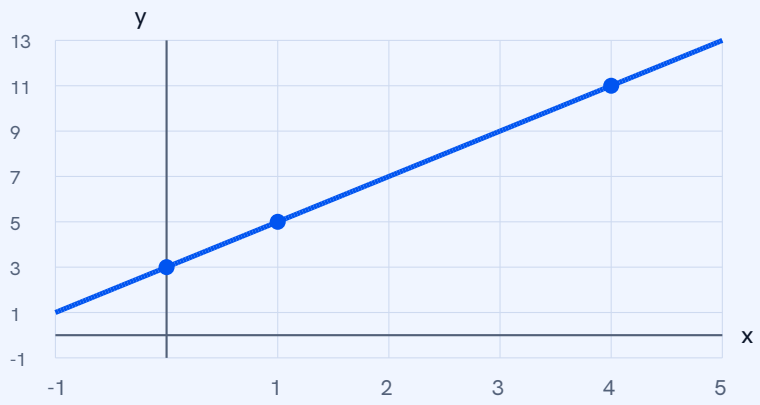
A linear function changes by the same amount whenever the input changes by the same amount. In y = mx + b, m is the output change per input unit and b is the output when x = 0. This description links an equation, a table, a graph and a real situation: all four should show the same rate and starting value.

Slope has a sign and units

Calculate slope as change in y divided by change in x, using the same point order in both differences. Positive slope means output rises as input rises; negative slope means it falls. A cost model may have units of dollars per hour, while a water-level model may have units of centimetres per minute.

Intercepts answer different questions

The y-intercept is the value at x = 0. The x-intercept is an input where y = 0, found by setting the output equal to zero. They need not both be meaningful in context: a business model may describe only nonnegative time, even though its line extends leftward on a graph.



y = 2x + 3

Points: (1, 5), (4, 11)

Rise = 6; run = 3

Slope = 6/3 = 2

y-intercept: (0, 3)

Illustrative teaching diagram. Values and examples are explained in the lesson.

THE ESSENTIAL IDEA

In y = mx + b, m is the rate of change and b is the starting value.

ALGEBRA / 02 BUILD THE METHOD

From idea to method

Build an equation from two points

For (1, 5) and (4, 11), slope is (11 - 5)/(4 - 1) = 2. Substitute either point into y = 2x + b: 5 = 2 + b, so b = 3. The resulting equation y = 2x + 3 must reproduce both supplied points.

Read a table by differences

For evenly spaced inputs, constant first differences in the outputs indicate a linear relationship. If input spacing varies, compare rates instead of raw output differences. An increase of 6 over three hours has the same rate as an increase of 2 over one hour.

Interpret a shifted input

In y = 2(x - 4) + 3, the number 3 is the output at x = 4, not at x = 0. Expanding gives y = 2x - 5, so the y-intercept is -5. Reading the form carefully is more reliable than treating every visible constant as an intercept.

WORKED EXAMPLE / A STARTING FEE AND A RATE

A hire service charges $7 initially and $4 per hour. Model the total cost C after h hours.

01 Set up the reasoning

The fixed charge is paid once, so it contributes 7. The hourly charge contributes 4h.

02 Apply the relationship

C = 4h + 7. After 3 hours, C = 12 + 7 = 19 dollars.

03 Interpret and check

An extra 2 hours adds 8 dollars regardless of the starting duration. That constant change is the meaning of slope.

Keep these distinctions handy

Rate	m = (y2 - y1)/(x2 - x1), for distinct x-values.
Starting output	b = y - mx, using any point on the line.
Zero output	Set y = 0 and solve; interpret the input in context.

ALGEBRA / 03 CONNECT & REMEMBER

See the deeper pattern

Compare models without calculating everything

For C = 4h + 7 and D = 6h + 1, the second option starts cheaper but grows faster. Set 4h + 7 = 6h + 1 to find h = 3. Both cost 19 dollars there. Before 3 hours D is cheaper; after 3 hours C is cheaper. The intersection separates the two pricing regimes.

ONE MORE WORKED APPLICATION

A service charges a $10 starting fee and $8 per hour. What is the cost for 11 hours, in dollars?

1. Hourly cost: 8 × 11 = 88.

2. Add the starting fee: 88 + 10 = 98.

Answer: 98

Mistakes to recognize

- Use the same ordering of points in the numerator and denominator.
- A vertical line cannot be written as a function y = mx + b.
- A steep line is not necessarily a larger rate if graph axes use different scales.

A quick understanding check

What does b represent in y = mx + b?

Explanation: The output at input zero, with the same units as y.

MORE PRACTICE IN YOUR DASHBOARD

Open prepvora.com/app/ and use the SAT question bank. Work on Algebra, then look for linear functions questions. Solve independently before reviewing the explanation. Use this guide for the theory; use the dashboard for a longer practice session.

Open your dashboard → | Explore the resource library →

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