ALGEBRA / 01 UNDERSTAND

Linear inequalities

An illustrated lesson ▪ Theory, reasoning and worked examples

A solution set rather than one value

An inequality compares quantities and usually allows many solutions. The statement $x < 4$ describes every real number smaller than 4; it is not just $x = 3$. The symbols < and > exclude equality, while ≤ and ≥ include it. A question about integers adds a separate restriction to the solution set.

Why a negative multiplier reverses order

On a number line, $2 < 5$. Multiplying both values by -1 places them at -2 and -5, and now $-2 > -5$. Reflection across zero reverses their order. This is why multiplying or dividing an inequality by a negative number reverses its comparison symbol.

The endpoint carries information

An open endpoint excludes the boundary; a filled endpoint includes it. A ray shows all values extending in one direction, not a single selected point. Always connect the algebraic symbol with the endpoint and direction so a correctly solved inequality is not graphed backward.

BOUNDARY + DIRECTION = THE SOLUTION SET



Illustrative teaching diagram. Values and examples are explained in the lesson.

THE ESSENTIAL IDEA

Inequalities describe a range of values rather than one solution.

ALGEBRA / 02 BUILD THE METHOD

From idea to method

Solve in a controlled sequence

Simplify both sides, collect variable terms, and isolate the variable. Addition and subtraction preserve the comparison direction. Multiplication or division by a positive number also preserves it. Only multiplication or division by a negative number reverses it.

Interpret compound restrictions

The statement $1 < x \leq 5$ means both $x > 1$ and $x \leq 5$ must hold. The overlap is the interval between 1 and 5, excluding 1 and including 5. By contrast, $x < 1$ or $x > 5$ includes either outside region. The words and and or change the set.

Translate context into a bound

At most means ≤; at least means ≥. If a ticket costs $8 and the budget is $35, $8n \leq 35$ gives $n \leq 4.375$. Since n counts tickets, the greatest permitted integer is 4. Rounding to the nearest integer is not the correct general method for a maximum under a budget.

WORKED EXAMPLE / A NEGATIVE COEFFICIENT

Solve $-3x + 6 \geq 15$ and describe the graph.

01 Set up the reasoning

Subtract 6: $-3x \geq 9$.

02 Apply the relationship

Divide by -3 and reverse the sign: $x \leq -3$.

03 Interpret and check

Draw a filled point at -3 and shade left. Test $x = -4$: $18 \geq 15$ is true. Test $x = 0$: $6 \geq 15$ is false.

Keep these distinctions handy

At least / at most	Inclusive bounds: ≥ / ≤. Use a filled endpoint.
Less / greater than	Strict bounds: < / >. Use an open endpoint.
Negative divisor	Reverse the comparison when dividing by a negative.

ALGEBRA / 03 CONNECT & REMEMBER

See the deeper pattern

Parameters require sign information

Dividing $ax < b$ by a is safe only after knowing the sign of a. If $a > 0$, then $x < b/a$; if $a < 0$, then $x > b/a$. If $a = 0$, inspect the constant comparison $0 < b$ instead. A symbolic coefficient is not automatically positive.

ONE MORE WORKED APPLICATION

What is the greatest integer x satisfying $9x + 12 < 75$?

1. Subtract 12 and divide by positive 9: $x < 7$.

2. The largest permitted integer is 6.

Answer: 6

Mistakes to recognize

- A strict endpoint cannot be included just because it is an integer.
- Do not reverse the sign when merely subtracting a negative number.
- Check a value inside and outside your proposed solution set.

A quick understanding check

Why does $2x < 8$ give a greatest integer of 3?

Explanation: The bound is $x < 4$; 4 is excluded, so 3 is the largest allowed integer.

MORE PRACTICE IN YOUR DASHBOARD

Open prepvora.com/app/ and use the SAT question bank. Work on Algebra, then look for linear inequalities questions. Solve independently before reviewing the explanation. Use this guide for the theory; use the dashboard for a longer practice session.

Open your dashboard → | Explore the resource library →

Original Prepvora explanations and examples. Scope: this topic, not the entire SAT syllabus. 2026-10-01 ▪ theory-v2